Chapter 7

Influence Lines for Beams

Instructional Objectives:

The objectives of this lesson are as follows:

- How to draw qualitative influence lines?
- Understand the behavior of the beam under rolling loads
- Construction of influence line when the beam is loaded with uniformly distributed load having shorter or longer length than the span of the beam.

Muller Breslau Principle for Qualitative Influence Lines

In 1886, Heinrich Muller Breslau proposed a technique to draw influence lines quickly. The Muller Breslau Principle states that the ordinate value of an influence line for any function on any structure is proportional to the ordinates of the deflected shape that is obtained by removing the restraint corresponding to the function from the structure and introducing a force that causes a unit displacement in the positive direction.

Let us say, our objective is to obtain the influence line for the support reaction at A for the beam shown in Figure 7.1.

Figure 7.1: Simply supported beam

First of all remove the support corresponding to the reaction and apply a force (Figure 7.2) in the positive direction that will cause a unit displacement in the direction of R_A. The resulting deflected shape will be proportional to the true influence line (Figure 7.3) for the support reaction at A.

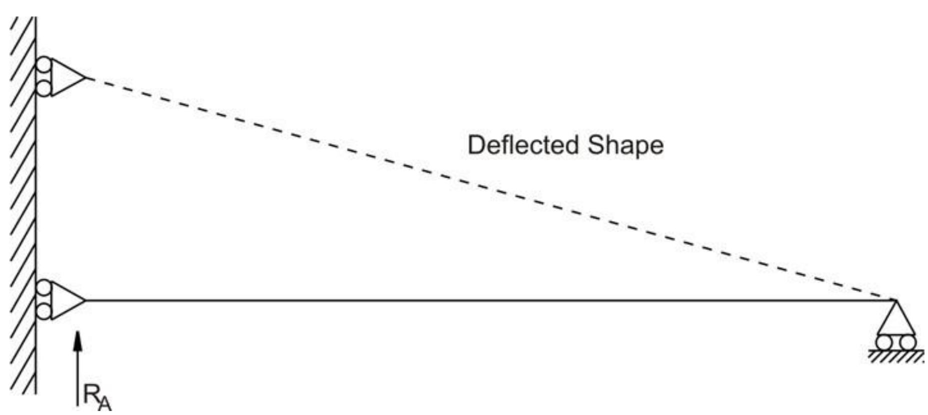

Figure 7.2: Deflected shape of beam

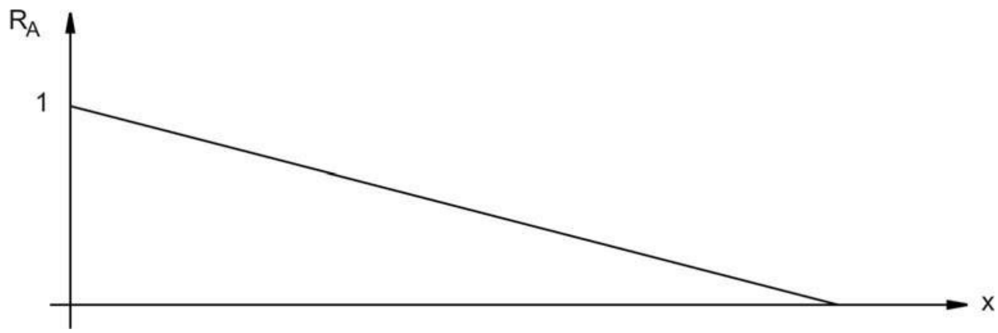

Figure 7.3: Influence line for support reaction A

The deflected shape due to a unit displacement at A is shown in Figure 7.2 and matches with the actual influence line shape as shown in Figure 7.3. Note that the deflected shape is linear, i.e., the beam rotates as a rigid body without any curvature. This is true only for statically determinate systems.

Similarly some other examples are given here.

Here we are interested to draw the qualitative influence line for shear at section C of overhang beam as shown in Figure 7.4.

Figure 7.4: Overhang beam

As discussed earlier, introduce a roller at section C so that it gives freedom to the beam in vertical direction as shown in Figure 7.5.

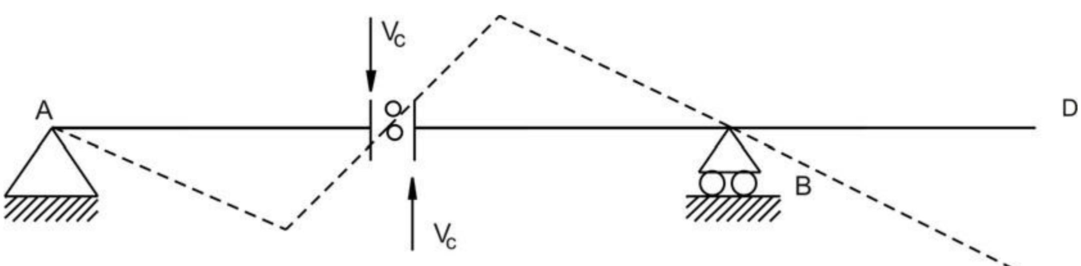

Figure 7.5: Deflected shape of beam

Now apply a force in the positive direction that will cause a unit displacement in the direction of V_C. The resultant deflected shape is shown in Figure 7.5. Again, note that the deflected shape is linear. Figure 7.6 shows the actual influence line, which matches with the qualitative influence.

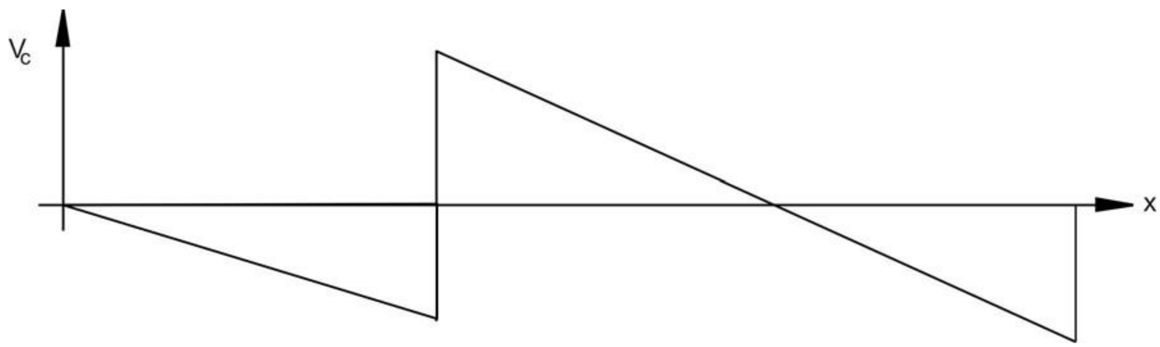

Figure 7.6: Influence line for shear at section C

In this second example, we are interested to draw a qualitative influence line for moment at C for the beam as shown in Figure 7.7.

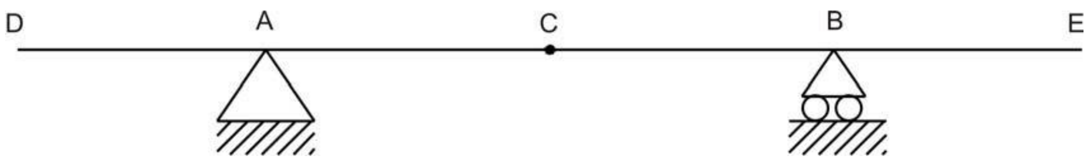

Figure 7.7: Beam structure

In this example, being our objective to construct influence line for moment, we will introduce hinge at C and that will only permit rotation at C. Now apply moment in the positive direction that will cause a unit rotation in the direction of M_c. The deflected shape due to a unit rotation at C is shown in Figure 7.8 and matches with the actual shape of the influence line as shown in Figure 7.9.

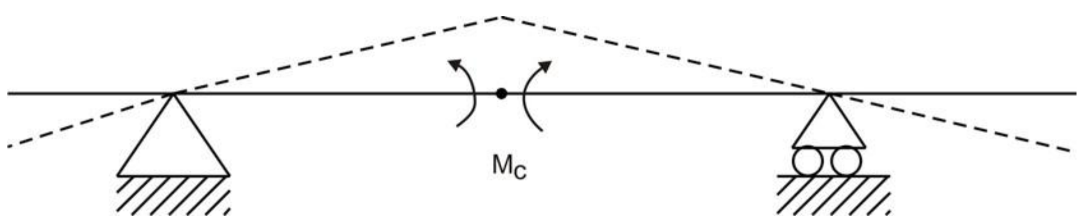

Figure 7.8: Deflected shape of beam

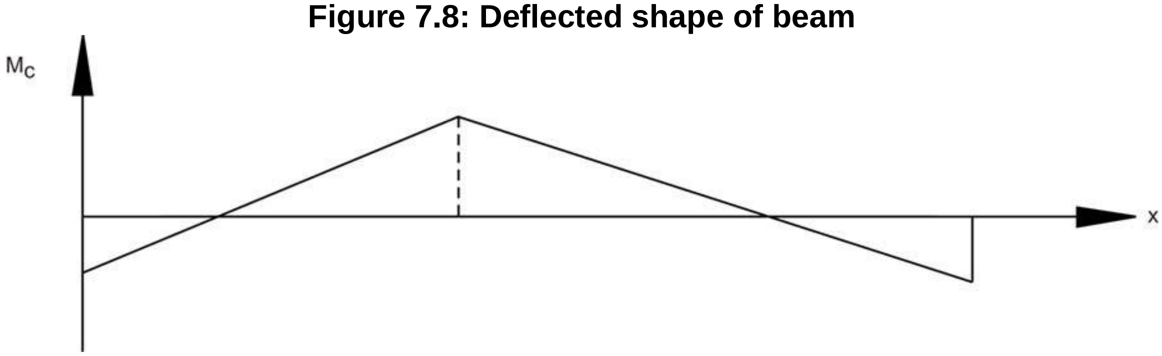

Figure 7.9: Influence line for moment at section C

Maximum shear in beam supporting UDLs

If UDL is rolling on the beam from one end to other end then there are two possibilities. Either Uniformly distributed load is longer than the span or uniformly distributed load is shorter than the span. Depending upon the length of the load and span, the maximum shear in beam supporting UDL will change. Following section will discuss about these two cases. It should be noted that for maximum values of shear, maximum areas should be loaded.

UDL longer than the span

Let us assume that the simply supported beam as shown in Figure 7.10 is loaded with UDL of w moving from left to right where the length of the load is longer than the span. The influence lines for reactions R_A, R_B and shear at section C located at x from support A will be as shown in Figure 7.11, 7.12 and 7.13 respectively. UDL of intensity w per unit length for the shear at supports A and B will be given by

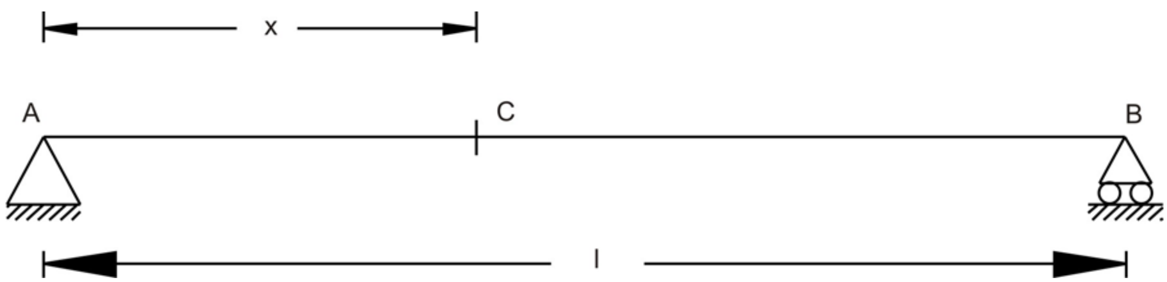

Figure 7.10: Beam Structure

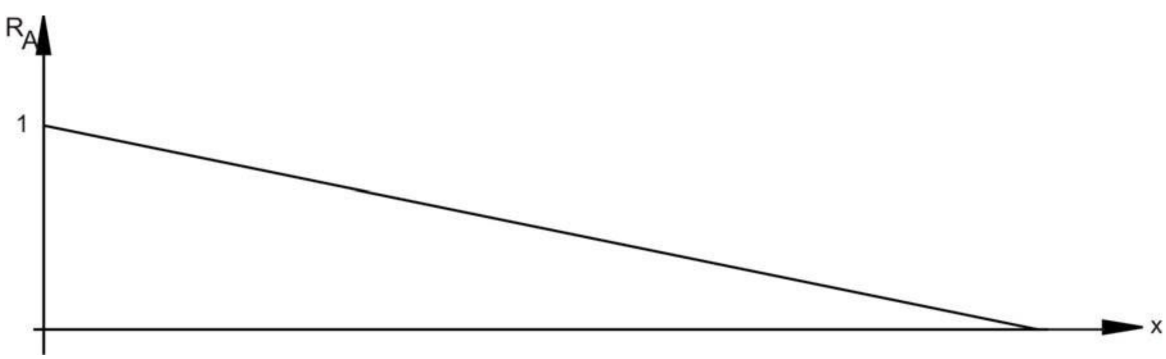

Figure 7.11: Influence line for support reaction at A

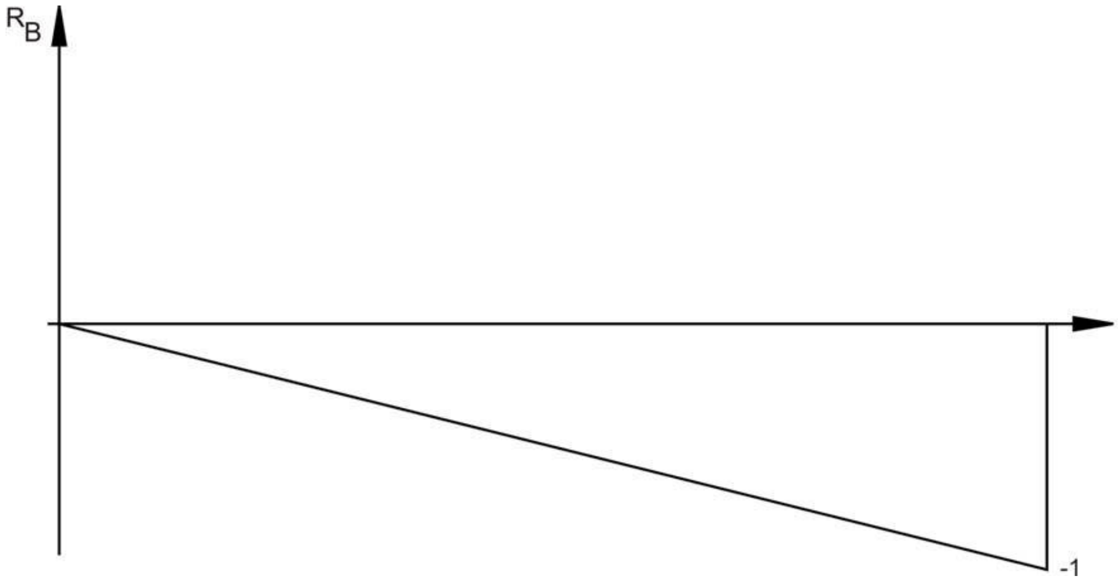

Figure 7.12: Influence line for support reaction at B

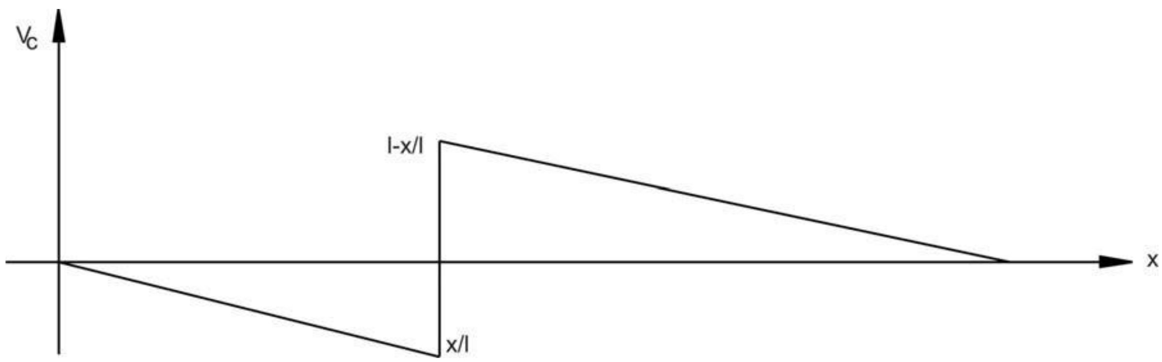

Figure 7.13: Influence line for shear at section C

$$R_A = w \times \frac{1}{2} \times l \times 1 = \frac{wl}{2}$$

$$R_B = -w \times \frac{1}{2} \times l \times 1 = \frac{-wl}{2}$$

Suppose we are interested to know shear at given section at C. As shown in Figure 7.13, maximum negative shear can be achieved when the head of the load is at the section C. And maximum positive shear can be obtained when the tail of the load is at the section C. As discussed earlier the shear force is computed by intensity of the load multiplied by the area of influence line diagram covered by load. Hence, maximum negative shear is given by

$$= -\frac{1}{2} \times x \times \frac{x}{l} \times w = -\frac{wx^2}{2l}$$

and maximum positive shear is given by

$$= \frac{1}{2} \times \left(\frac{l-x}{l} \right) \times (l-x) \times w = -\frac{w(l-x)^2}{2l}$$

UDL shorter than the span

When the length of UDL is shorter than the span, then as discussed earlier, maximum negative shear can be achieved when the head of the load is at the section. And maximum positive shear can be obtained when the tail of the load is at the section. As discussed earlier the shear force is computed by the load intensity multiplied by the area of influence line diagram covered by load. The example is demonstrated in previous lesson.

Maximum bending moment at sections in beams supporting UDLs.

Like the previous section discussion, the maximum moment at sections in beam supporting UDLs can either be due to UDL longer than the span or due to ULD shorter than the span. Following paragraphs will explain about computation of moment in these two cases.

UDL longer than the span

Let us assume the UDL longer than the span is traveling from left end to right hand for the beam as shown in Figure 7.14. We are interested to know maximum moment at C located at x from the support A. As discussed earlier, the maximum bending moment is given by the load intensity multiplied by the area of influence line (Figure 7.15) covered. In the present case the load will cover the completed span and hence the moment at section C can be given by

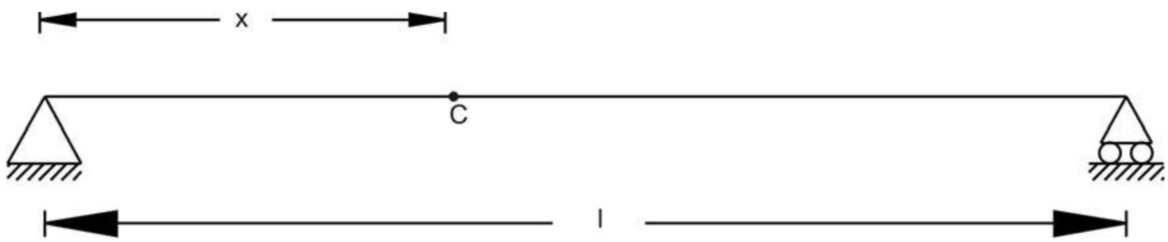

Figure 7.14: Beam structure

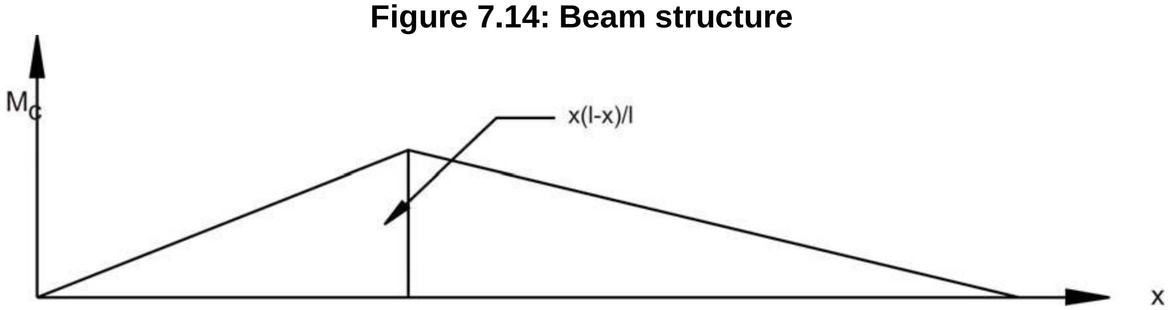

Figure 7.15: Influence line for moment at section C

$$w \times \frac{1}{2} \times l \times \frac{x(l-x)}{l} = -\frac{wx(l-x)}{2}$$

Suppose the section C is at mid span, then maximum moment is given by

$$\frac{w \times \frac{l}{2} \times \frac{l}{2}}{2} = \frac{wl^2}{8}$$

UDL shorter than the span

As shown in Figure 7.16, let us assume that the UDL length y is smaller than the span of the beam AB. We are interested to find maximum bending moment at section C located at x from support A. Let say that the mid point of UDL is located at D as shown in Figure 7.16 at distance of z from support A. Take moment with reference to A and it will be zero.

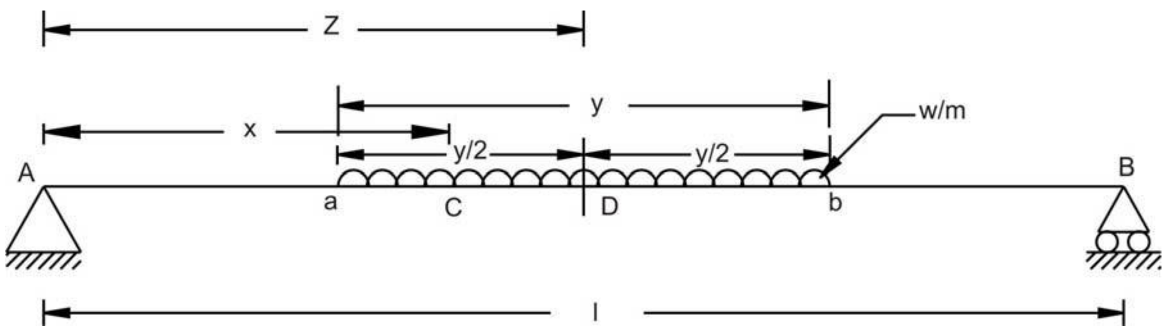

Figure 7.16: Beam loaded with UDL shorter in length than span

Hence, the reaction at B is given by

$$R_B = w \times y \times \frac{z}{l} = -\frac{wx(l-x)}{2}$$

And moment at C will be

$$M_C = R_B(l-x) - \frac{w}{2}(z + \frac{y}{2} - x)^2$$

Substituting value of reaction B in above equation, we can obtain

$$M_C = \frac{wyz}{l}(l-x) - \frac{w}{2}(z + \frac{y}{2} - x)^2$$

To compute maximum value of moment at C, we need to differentiate above given equation with reference to z and equate to zero.

$$\frac{dM_c}{dz} = \frac{wy}{l}(l - x) - w(z + \frac{y}{2} - x) = 0$$

Therefore,

$$\frac{y}{l}(l - x) = (z + \frac{y}{2} - x)$$

Using geometric expression, we can state that

$$\frac{ab}{AB} = \frac{Cb}{CB}$$

$$\therefore \frac{CB}{Cb} = \frac{AB}{ab} = \frac{AB - CB}{ab - Cb} = \frac{AC}{aC}$$

$$\therefore \frac{aC}{Cb} = \frac{AC}{CB}$$

The expression states that for the UDL shorter than span, the load should be placed in a way so that the section divides it in the same proportion as it divides the span. In that case, the moment calculated at the section will give maximum moment value.

Closing Remarks

In this lesson we studied how to draw qualitative influence line for shear and moment using Muller Breslau Principle. Further we studied how to draw the influence lines for shear and moment when the beam is loaded with UDL. Here, we studied the two cases where the UDL length is shorter or longer than span. In the next lesson the effect of two or more than two concentrated loads moving on the beam will be studied.

Suggested Text Books for Further Reading

- Hibbeler, R. C. *Structural Analysis*, Pearson Education (Singapore) Pvt. Ltd., Delhi, ISBN 81-7808-750-2

- Junarkar, S. B. and Shah, H. J. *Mechanics of Structures – Vol. II*, Charotar Publishing House, Anand.